Can hadrons survive the chiral phase transition?

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Abstract

We study mesonic and baryonic correlation functions in the temporal directions in the vicinity of the critical temperature $T \simeq T_c$ using the instanton liquid model, in which chiral symmetry restoration is driven by the formation of instanton-antiinstanton molecules. Although we find the signals for all hadronic poles to be drastically reduced, some hadronic states seem to survive the phase transition, as loosely bound $U(N_f)\times U(N_f)$ chiral multiplets in the plasma phase. These states are the $0^+ - 0^-$ mesons $(\pi, \sigma, \eta', \delta)$ and the $N(\frac{1}{2}^+) - N(\frac{1}{2}^-)$ baryons.

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1 Introduction

It was conjectured long ago [1] and later confirmed by a large number of lattice simulations (see the review [2]) that at high temperatures QCD undergoes a transition to a new phase, the quark-gluon plasma, in which (i) chiral symmetry is restored and (ii) color charges are *screened* [3] rather than *confined*. It is still unknown whether in the real world (for physical values of quark masses) this transition is a true singularity or just a rapid crossover. However, it is well established that qualitative changes happen in a relatively narrow region $\Delta T \simeq 10\,\mathrm{MeV} \ll T_c \simeq 150$ MeV. In this region, for example, the energy density grows by about an order of magnitude [4].

While physical phenomena below and above this region can (at least semi-quantitatively) be understood in terms of weekly interacting gases of hadrons or quark and gluons, respectively, it remains unclear how one should treat the phenomena close to T_c. *If* there is no strict phase transition and *if* the time scale is long enough to support a state in thermodynamic equilibrium, one may expect to see matter in which the excitation spectrum gradually changes from hadronic states to quarks and gluons. In terms of spectral densities this means that hadronic poles gradually dissolve and quark (or gluon) cuts move toward smaller energies. This is the phenomenon we are going to discuss in this paper.

For completeness, let us mention two other possibilities. *If* there is a first order transition and the relaxation time is sufficiently long, a real mixed phase appears, in which the two different phases coexist. The space and time scale of the domains (or "bubbles") is then determined by the production dynamics [5], mostly governed by the surface tension σ. Available lattice simulations seem to indicate that it is surprisingly small [6]. Therfore, tunneling is relatively easy and one would expect that heavy ion collisions produce rather *well mixed* matter.

Another special case is the second order transition, in which the system is governed by the dynamics of long-wavelength fluctuations of the order parameter. The chiral phase transition in QCD with two massless quark flavors is such an example, although the real world might be closer to another second order transition, corresponding to the boundary

of the three flavor first order transition. Whether or not the critical behavior of the two flavor phase transition is identical to the O(4) Heisenberg magnet [7] or given by mean field analysis [8] is currently under debate.

At zero temperature there is strong support for the idea that instantons dominate the physics of chiral symmetry breaking and and explain many properties of light hadrons while confinement plays a relatively small role [9, 10]. It is quite plausible that this is also true near the phase transition. At the moment, there are direct lattice measurements (in quenched QCD) indicating that the instanton density near the phase transition is very similar to its value at $T = 0$ [11], in agreement with the arguments given in [12]. In this letter, we will assume that there is a sizeable number of instantons present near T_c and study the implications of this scenario for the survival of hadronic modes in the "mixed phase" region.

The mechanism of the chiral phase transition in the instanton liquid is related to a rearrangement of the instantons [13, 14], going from a disordered, random liquid to a phase of correlated instanton-antiinstanton molecules[1]. In [14] we have developed a simple model in order to describe phenomena in the vicinity of the phase transition. In this model we neglect the finite width ΔT of the transition and consider the system at a fixed temperature $T = T_c = 150$ MeV. The relevant parameter in this case is not T but f, the fraction of instantons that are paired in "molecules". As f is varied from 0 to 1 one passes through the mixed phase region, going from the chirally broken phase at $f = 0$ to the restored phase at $f = 1$. All observables are calculated as a function of the parameter f. For example, at $f \to 1$ the quark condensate $\langle \bar{q}q \rangle \to 0$, the energy density grows, etc. In [14] we showed that instantons generate non-perturbative interactions even when chiral symmetry is restored. In particular, we were able to reproduce lattice results for spacelike screening masses near the transition.

Before we study the behavior of hadrons, we would like to remind the reader about a few properties of correlation functions at finite temperature [15]. Since Lorentz invari-

[1] Similar attempts at understanding the deconfinement transition usually focus on the rearrangement of monopole loops at T_c [16], but the underlying physics is not very well understood at the moment.

ance is broken, there are two different kinds of correlators, which depend either on the spatial distance x or the temporal separation τ. In the case of spacelike separation one can go to large x and filter out the lowest exponents known as "screening masses". Lattice measurements of the screening masses [17] have revealed an interesting spectroscopy of "quasi-bound states". Additional evidence for significant deviations from free quark propagation was given in [18], where the corresponding "wave functions" were measured. These wave functions are localized even at very large temperatures, a result that created speculations about the survival of hadronic bound states at large T [19, 20]. However, as discussed in detail in [21, 22], although these states indeed exist at *any* T, they have nothing to do with real bound states. As $T \to \infty$ QCD becomes dimensionally reduced and these "states" form the spectrum of $d = 3$ QCD.

In order to look for *real* bound states, on has to study *temporal* correlation functions. However, at finite temperature temporal correlators can only be measured for distances $\tau < 1/2T$ (about 0.6 fm at $T = T_c$), so one can not trivially filter out the ground state. For that reason, temporal correlators were generally believed to be dominated by perturbative phenomena and fairly useless for the study of hadronic states: but with some effort and skill some useful information about the lowest excitations can still be extracted. On the lattice that was attempted by Boyd et al. [23], who in particular observed a dramatic difference between the pion and rho channels.

In this paper we report our results for a number of temporal correlation functions in the "cocktail model" mentioned above. The correlation functions are calculated from the quark propagator in the instanton liquid. The details of this calculations can be found in [14]. Here we only note a few important properties of this propagator. First, it of course satisfies the correct antiperiodic boundary conditions in the finite temperature euclidean box and reduces to the free propagator at short distances. Furthermore, we numerically invert the dirac operator in the basis spanned by the instanton zero modes. This means that the propagator contains the instanton induced interaction to all orders. When chiral symmetry is broken, this is the 't Hooft effective interaction, when the symmetry is

restored, it is the effective interaction induced by molecules discussed in [14]. In the following, we will study correlation functions

$$\Pi(\tau) \;=\; <\bar{q}\Gamma q(\tau)\bar{q}\Gamma q(0)>, \tag{1}$$

where $\bar{q}\Gamma q$ is a mesonic current with the quantum numbers of the gamma and isospin matrix Γ. Correlators with baryonic quantum numbers are defined analogously. We will normalize the correlation functions to the corresponding free correlators at the same temperature,

$$\Pi_0(\tau) \;=\; \mathrm{Tr}[\Gamma S_0(T,\tau)\Gamma S_0(T,-\tau)]\,, \tag{2}$$

$$S_0(T,\tau) \;=\; \frac{\gamma_0}{2\pi^2}\sum_{n=-\infty}^{\infty}\frac{(-)^n}{(\tau+n/T)^3}\,. \tag{3}$$

If the ratio $R=\Pi(\tau)/\Pi_0(\tau)$ is larger than one, we will refer to the correlator as attractive, while deviations down imply repulsive interactions.

Fig.1a shows the pseudoscalar correlator with the quantum numbers of the pion. At $f=0.25$ there is a very strong signal which melts down as f approaches one. Remarkably though, it is *not* proportional to $(1-f)$ as one might naively expect: in particular, there is a significant signal at $(1-f)=0.05$. Furthermore, even as chiral symmetry is restored ($f=1$, see fig.2c) the correlator is still larger than the perturbative one, and the excess can still be interpreted in terms of a hadronic bound state supplementing the background from free quark propagation.

This behaviour can be contrasted to the one shown in fig.1b, for the scalar-isovector (δ meson) correlator. At small f one finds a strongly decaying correlation function, indicative of a strongly repulsive effective interaction, or the absence of light states in this channel. As $f\to 1$ this repulsion disappears, and the the correlator grows with f, until at $f=1$ the correlator is larger than the perturbative one. Furthermore, one finds that in this case the delta correlation function is equal to the pion correlator. Since the chiral partner of the pion is the sigma meson, not the delta, this is a sign of $U_A(1)$ restoration [24]. From chiral symmetry one then concludes that all scalar mesons $(\sigma,\pi,\eta',\delta)$ are degenerate. We have checked this point by calculating the relevant disconnected diagramms for the η' and

σ correlation functions. Note that $U_A(1)$ is restored not because the anomaly disappears, but because the formation of molecules implies that topological charge is screened over very short distances.

In order to study the properties of the correlation functions in a more quantitative fashion, we have fitted the mesonic correlators using the following parametrization

$$\Pi(\tau) = \lambda^2 D(T, M, \tau) + \mathrm{Tr}[\Gamma S_m^V(T,\tau)\Gamma S_m^V(T,-\tau)]\,, \tag{4}$$

as well as an analogous formula for baryonic correlators. The first term corresponds to the propagation of a meson resonance with mass M and coupling constant λ, while the second term describes the quark-antiquark continuum. Here, $S_m^V(T,\tau)$ is the vector (chiral even) part of the temporal propagator of a free quark with mass m at temperature T. As m goes to zero, the quark-antiquark threshold moves down to zero energy and the correlation function becomes perturbative.

Applying this parametrization to the pion correlator we find that both the resonance mass and the chiral quark mass at $f = 1$ are small (consistent with zero). This allows us to put a lower limit on the pseudosclar coupling for which we find $\lambda_\pi = 1.8 \pm 0.3\,\mathrm{fm}^{-2}$. The product of λ_π and the pion decay constant f_π can be determined from the off-diagonal pseudoscalar-axialvector (γ_5-$\gamma_0\gamma_5$) correlation function. This provides a very clean measurement, since free quarks do not contribute to this correlator. From the data shown in fig.1c and the coupling constants listed in table 1 one clearly observes how f_π goes to the zero as chiral symmetry is restored while λ_π remains finite. To study this point in more detail we have calculated the $f = 1$ correlators for various values of the "valence quark" mass (the mass that enters the quark propagator). We find that $\lambda_\pi f_\pi$ scales as the quark mass while λ_π is finite in the chiral limit. The $f = 1$ curves shown in fig.2c are calculated for $m_u = m_d = 4$ MeV, all other curves are for $m_u = m_d = 20$ MeV.

Proceeding from the scalar channels to other cases (vector mesons, baryons etc.) one knows in advance that for most of them the splitting between the ground state and excited states is much smaller, and (at the distances considered) one finds relatively small deviations from free quark propagation. At zero temperature one can circumvent this

problem by considering off-diagonal correlators, in which free quark propagation does not contribute. In addition to the pseudosclar-axialvector correlator mentioned above, we have also calculated the vector-tensor correlators γ_ν-$\sigma_{\mu\nu}$ (fig.2d). Again, the correlator can be described nicely in terms of the propagation of a single rho meson bound state, without a free quark continuum. For a definition of the corresponding coupling constant c_ρ, see [9]. However, as chiral symmetry is restored ($f = 1, m \to 0$), this signal disappears. One can check that the vanishing of the vector-tensor correlator is indeed a consequence of chiral symmetry restoration, and the small signal that we observe is consistent with perturbative mass corrections. The diagonal $\vec{\gamma}$-$\vec{\gamma}$ rho meson correlator in the chirally restored phase is shown in fig.2c. There is no evidence for a resonance. Indeed, the correlator is slightly suppressed with respect to the perturbative one. This suppression is consistent with a chiral quark mass $m \simeq 85$ MeV. The diagonal γ_4-γ_4 correlator shows a very small enhancement, numerically smaller but in qualitative agreement with the molecular interaction discussed in [14].

Let us now proceed to baryon correlators. As explained in detail in [9], there are 6 nucleon and 4 delta correlation functions that can be constructed out of the two Ioffe currents for the nucleon and the unique delta current. Half of these correlators, $\Pi^N_{1,3,5}$ and $\Pi^\Delta_{1,3}$ in the notation of [9] are chiral odd and have to vanish as chiral symmetry is restored. $\Pi^N_{2,4}$ as well as $\Pi^\Delta_{2,4}$ are chiral even and may show resonance signals even as chiral symmetry is restored. Π^N_6, the chiral even, off-diagonal correlator of the two Ioffe currents is special. Chiral symmetry does not require it to vanish, but since the $U_A(1)$ transformation properties of the two currents do not match, it is an order parameter for $U_A(1)$ symmetry breaking.

The chiral odd correlators Π^N_1 and Π^Δ_1 are shown in figs.2a,c. These correlators receive no contribution from free quark propagation and one can observe clear resonance signals that die away as $f \to 1$. Note that there are several possible explanations for this behavior. If one includes the lowest postive and negative parity resonances in the spectral function, then the correlator Π^N_1 is proportional to $\beta_{N1} = \lambda^2_{1N(\frac{1}{2}^+)} m_{N(\frac{1}{2}^+)} - \lambda^2_{1N(\frac{1}{2}^-)} m_{N(\frac{1}{2}^-)}$, where

		RILM, $T=0$	$f=0.25$	$f=0.50$	$f=0.75$	$f=0.95$	$f=1.00$
λ_π^2	[fm^{-4}]	44±10	99±6	61±3	49±3	22±4	1.8±0.3
λ_δ^2	[fm^{-4}]	<0.1	<0.1	<0.1	<0.1	<0.1	1.8±0.3
$\lambda_\pi f_\pi$	[fm^{-3}]	3.7±0.5	1.7±0.15	1.2±0.12	0.8±0.06	0.3±0.06	<0.1
c_ρ	[fm^{-3}]	6±1	1.7±0.24	1.5±0.24	0.8±0.06	0.4±0.06	<0.06
β_{N1}	[fm^{-7}]	80±17	150±26	109±25	63±25	25±15	<10
$\beta_{\Delta 1}$	[fm^{-7}]	130±35	122±15	85±15	63±13	29±10	<8
β_{N2}	[fm^{-6}]	16±2					1.1±0.7
β_{N6}	[fm^{-7}]		930±60	730±50	500±50	190±40	<20

Table 1: Coupling constants extracted from the parametrization of the temporal correlation functions decsribed in the text, versus the fraction of instanton molecules, f. The coupling constants are also defined in the text, for more details see [9]. For comparison, we also show our zero temperature results for the completely random ensemble ($f=0$).

$\lambda_{1N(\frac{1}{2}^+)}$ is the coupling of the first Ioffe current to the positive parity nucleon, etc. In this expression, we have suppressed the resonance propagators, and there is an analogous result for the delta. This means that the correlator can vanish either because the masses go to zero or the resonances decouple, or because the parity partners become degenerate and their contributions to the correlator cancel.

The chiral even nucleon and delta correlators $\Pi_2^{N,\Delta}$ in the chirally restored phase are shown in fig.2c. One observes a small attractive signal in the nucleon case, while no such signal is present for the delta. In this case, the resonance coupling is proportional to $\beta_{N2}=\lambda^2_{1N(\frac{1}{2}^+)}+\lambda^2_{1N(\frac{1}{2}^-)}$, so the observed signal in the nucleon channel can be interpreted in terms of nucleon state with non-vanishing coupling constant, possibly with an equal contribution from the negative parity state. As was the case in the rho meson channel, the repulsion observed in the delta channel can be explained in terms of a chiral quark mass $m\simeq 85$ MeV.

Finally, the off-diagonal nucleon correlator Π_6^N is shown in in fig.2b. This correlator shows a very large signal at $f=0$ which completely disappears as $f\to 1$. This is a clear indication of $U_A(1)$ restoration and, most likely, parity doubling in the nucleon spectrum. In this case the resonance contribution is $\beta_{N6}=\lambda_{1N(\frac{1}{2}^+)}\lambda_{2N(\frac{1}{2}^+)}+\lambda_{1N(\frac{1}{2}^-)}\lambda_{1N(\frac{1}{2}^-)}$, so $U_A(1)$ restoration implies a relation between the various couplings.

We have summarized our results for the coupling constants in table 1. For comparison we have also included our results at zero temperature. Although one should keep in mind that the resonance couplings at $T = 0$ and $T = T_c$ are determined at different distances, a rough comparison can still be made. Most channels simply show a gradual decrease of the coupling constants, except for the pion and the nucleon, where the couplings at $T = 150$ MeV and $f = 0.25$ are actually bigger than the $T = 0$ values. Most of this effect is probably an artefact due to the use of a completely random ensemble at finite temperature. This ensemble also gives a quark condensate which is somewhat larger than the $T = 0$ value. We will study this point in more detail in a forthcoming publication, where results from fully interacting ensembles will be presented [25]. Comparing the squared pion coupling at $T = 0$ to its value at $f = 1$ one finds that it drops by about a factor 20: assuming the coupling to be proportional to $|\psi(0)|^2$, the probability to find a quark and an antiquark at the same point, which is inversely proportional to the volume, one concludes that the radius of the pion increases by nearly a factor 3.

In summary, we have presented an analysis of temporal correlation functions for mesons and baryons in the instanton vacuum. In this model, chiral symmetry restoration is due to the formation instanton-antiinstanton molecules. In agreement with lattice results [23], we have found significant non-perturbative effects for scalar-pseudoscalar mesons even in the chirally symmetric phase. Unfortunately, much better accuracy is needed to accurately determine the mass or even the width of these states and settle the question whether they are just some broad enhancement in the spectrum or fairly narrow resonaces. We also find an attractive interaction in the nucleon channel, while it is abesent in the case of the delta. It may indicate that some hadronic states are not completely dissolved at the phase transition, but survive as weakly bound $U(N_f)\times U(N_f)$ chiral multiplets.

References

[1] J. C. Collins, M. J. Perry, Phys. Rev. Lett. **34**, 1353 (1975)

[2] F. Karsch, Nucl. Phys. (Proc. Suppl.) **B 34** (1994) 63. C. DeTar, Quark gluon plasma in numerical simulations of lattice QCD, To appear in "Quark Gluon Plasma 2", R. Hwa, ed., World Scientific (1995)

[3] E. Shuryak, Zh. Eksp. Teor. Fiz. (Sov. Phys. JETP), **74** (1978) 408. J. Kapusta, Nucl. Phys. **B148** (1979) 461.

[4] T. Blum, L. Kärkkäinnen, D. Toussaint, S. Gottlieb, Phys. Rev. **D51**, 5133 (1995)

[5] L. P. Csernai, J. I. Kapusta, Phys. Rev. Lett. **69**, 737 (1992)

[6] K. Kajantie, L. Kärkkäinen, K. Rumukainen, Nucl. Phys. **B333**, 100 (1990)

[7] R. Pisarski, F. Wilczek, Phys. Rev. **D 29**, 338 (1983)

[8] A. Kocic, J. Kogut, Phys. Rev. Lett. **74**, 3109 (1995)

[9] E. Shuryak, J. Verbaarschot, Nucl. Phys. **B410**, 55 (1993); T. Schäfer, E. V. Shuryak, J. J. M. Verbaarschot, Nucl. Phys. **B412**, 143 (1994)

[10] M. C. Chu, J. M. Grandy, S. Huang , J. W. Negele, Phys. Rev. **D49**, 6039 (1994)

[11] M. C. Chu, S. Schramm, Phys. Rev. **D51**, 4580 (1995)

[12] E. V. Shuryak, M. Velkovsky, Phys. Rev. **D50**, 3323 (1994)

[13] M.-E.Ilgenfritz, E. V. Shuryak, Phys. Lett. **B325**, 263 (1994)

[14] T. Schäfer, E. V. Shuryak, J. J. M. Verbaarschot, Phys. Rev. **D51**, 1267 (1995)

[15] E. V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993)

[16] T. Suzuki, Nucl. Phys. (Proc. Supp.) **B30**, 176 (1993)

[17] C. De Tar, J. Kogut, Phys. Rev. **D36**, 2828 (1987)

[18] C. Bernard et al., Phys. Rev. Lett. 68, 2125 (1992)

[19] C. deTar, Phys. Rev. **D37**, 2328 (1988)

[20] I. Zahed, Proc. of the int. conf. on "Thermal Field Theories", H. Ezawa et al., eds., Elsevier (1991)

[21] V. Koch, E. V. Shuryak, G. E. Brown, A. D. Jackson, Phys. Rev. **D46**, 3169 (1992)

[22] T. H. Hansson, I. Zahed, Nucl. Phys. **B374**, 277 (1992)

[23] G. Boyd, S. Gupta, F. Karsch, E. Laerman, Z. Phys. **C64**, 331 (1994)

[24] E. V. Shuryak, Comm. Nucl. Part. Phys. **21**, 235 (1994)

[25] T. Schäfer, E. V. Shuryak, J. J. M. Verbaarschot, Hadronic correlation functions in the interacting instanton model, preprint, SUNY-NTG-95-23

figure captions

figure 1 Isovector meson correlation functions in the temporal direction, at $T = 150$ MeV, normalized to free quark propagation. Fig.1(a) shows the pseudoscalar (pion) correlator, (b) the scalar isovector (delta), (c) the pseudoscalar-axialvector correlator and (d) the vector-tensor one. All correlators are shown vor various fractions of instanton molecules: open squares are $f = 0.25$, open hexagons $f = 0.50$, skeletal stars $f = 0.75$, crosses $f = 0.95$ and open stars $f = 1.00$.

figure 2 Temporal correlation functions as in fig.1. Fig.2(a) shows the chiral odd nucleon correlator Π_1^N, (b) the chiral even but $U_A(1)$ breaking correlator Π_6^N, (c) the chiral even nucleon and delta correlators $\Pi_2^{N,\Delta}$ compared to the pion and rho at $f = 1$ only, and (d) the chiral odd delta correlator Π_1^Δ.

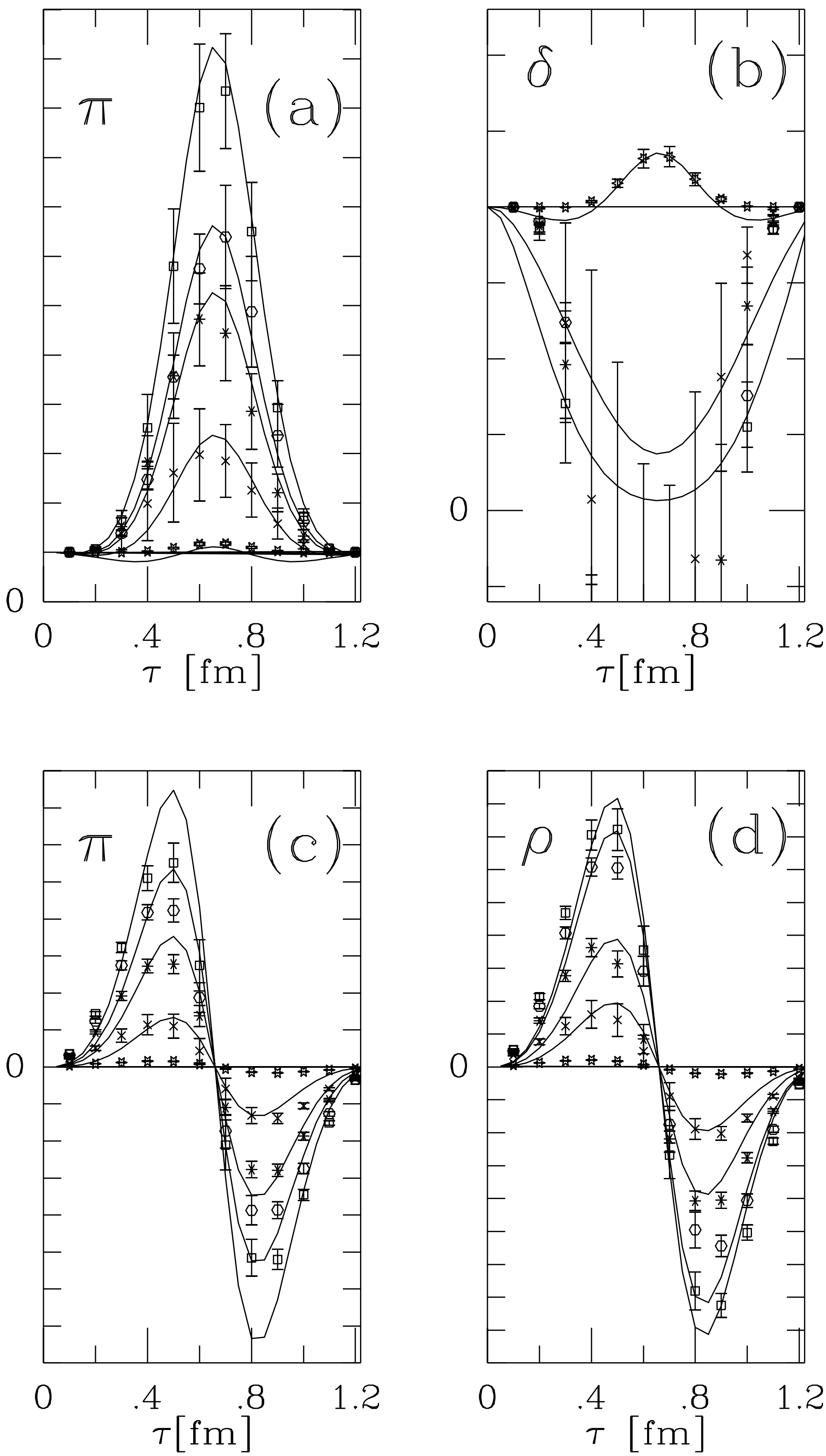
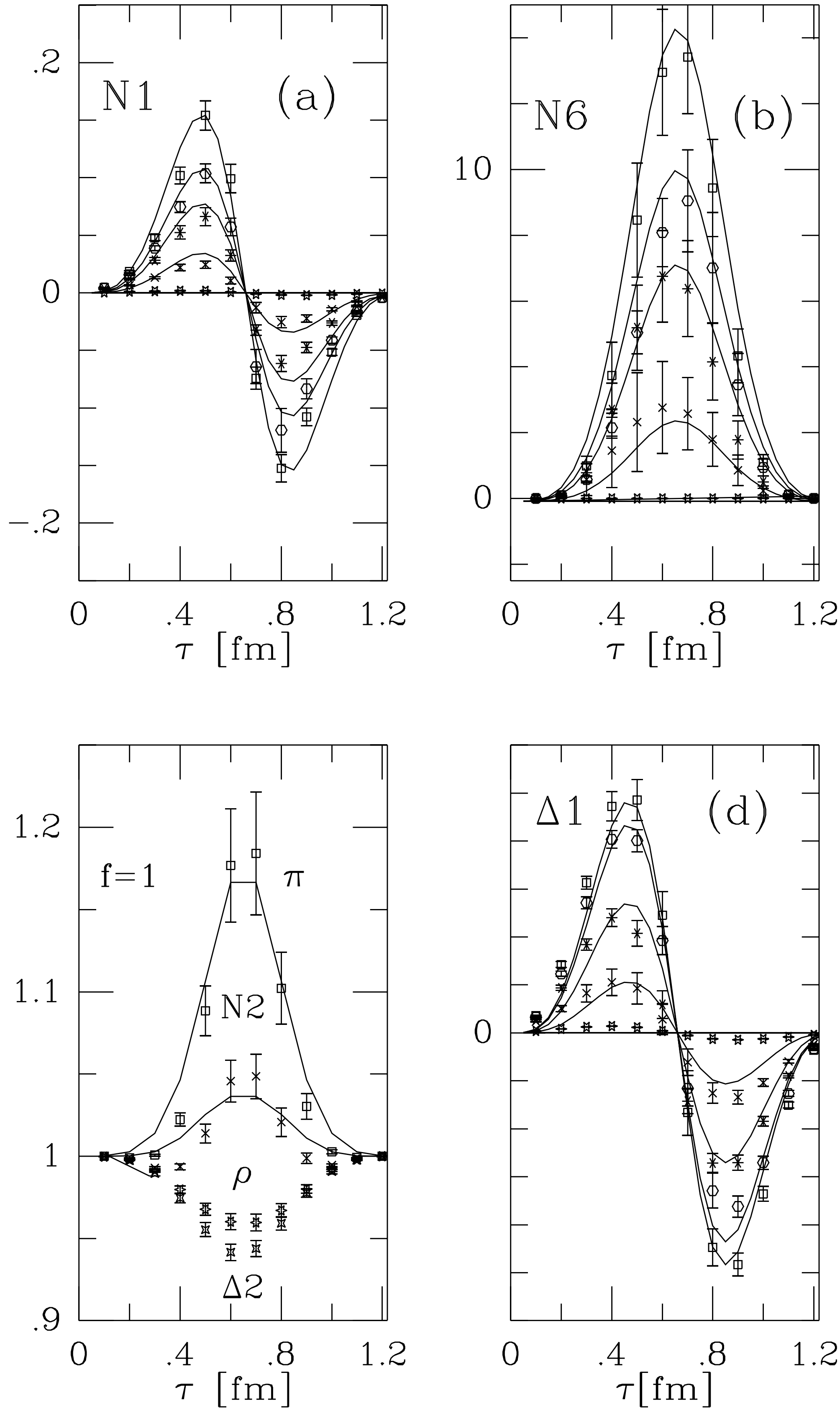


Figure 1:



N1
(a)
.2
0
−.2
0
.4
.8
1.2
τ [fm]
N6
(b)
10
0
0
.4
.8
1.2
τ [fm]
f=1
π
N2
ρ
Δ2
1.2
1.1
1
.9
0
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.8
1.2
τ [fm]
Δ1
(d)
0
0
.4
.8
1.2
τ[fm]

Figure 2: